Determine whether or not $\vec{\mathbf{F}}$ is a conservative vector field. If it is, find a function f such that $\vec{\mathbf{F}} = \nabla f$.

1) $\vec{\mathbf{F}}(x, y) = (6x + 5y)\,\mathbf{i} + (5x + 4y)\,\mathbf{j}$

$$\boxed{\vec{\mathbf{F}} \text{ is conservative, } f(x, y) = 3x^2 + 5xy + 2y^2 + K}$$

2) $\vec{\mathbf{F}}(x, y) = (x^3 + 4xy)\,\mathbf{i} + (4xy - y^3)\,\mathbf{j}$

$$\boxed{\vec{\mathbf{F}} \text{ is not conservative.}}$$

3) $\vec{\mathbf{F}}(x, y) = (1 + 2xy + \ln x)\,\mathbf{i} + x^2\,\mathbf{j}$

$$\boxed{\vec{\mathbf{F}} \text{ is conservative, } f(x, y) = x^2 y + x \ln x + K}$$

Find a function f such that $\vec{\mathbf{F}} = \nabla f$ and use it to evaluate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ along the given curve C.

4) $\vec{\mathbf{F}}(x, y) = y\,\mathbf{i} + (x + 2y)\,\mathbf{j}$; C is the upper semicircle that starts at $(0, 1)$ and ends at $(2, 1)$.

$$\boxed{f(x, y) = xy + y^2, \quad \int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = 2}$$

5) $\vec{\mathbf{F}}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy + 2z)\,\mathbf{k}$; C is the line segment from $(1, 0, -2)$ to $(4, 6, 3)$.

$$\boxed{f(x, y, z) = xyz + z^2, \quad \int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = 77}$$

6) $\vec{\mathbf{F}}(x, y, z) = (2xz + y^2)\,\mathbf{i} + 2xy\,\mathbf{j} + (x^2 + 3z^2)\,\mathbf{k}$; $C:\ x = t^2,\ y = t + 1,\ z = 2t - 1,\quad 0 \le t \le 1$

$$\boxed{f(x, y, z) = x^2 z + xy^2 + z^3, \quad \int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = 7}$$

7) $\vec{\mathbf{F}}(x,y,z) = y^2\cos z\,\mathbf{i} + 2xy\cos z\,\mathbf{j} - xy^2\sin z\,\mathbf{k}$; C: $\vec{\mathbf{r}}(t) = t^2\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}, \quad 0 \le t \le \pi$

$$\boxed{f(x,y,z) = xy^2\cos z, \quad \int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = 0}$$

8) $\vec{\mathbf{F}}(x,y,z) = \left(15z^2e^{3x-2y} - 12x^2y\right)\mathbf{i} + \left(-10z^2e^{3x-2y} - 4x^3 - \frac{2}{3}y^{-1/3}\right)\mathbf{j} + \left(10ze^{3x-2y} + 3\right)\mathbf{k}$

C: $\vec{\mathbf{r}}(t) = \left(2t^2\right)\mathbf{i} + \left(t^3 + t^2 + 1\right)\mathbf{j} + \left(-3t + 4\right)\mathbf{k}, \quad 0 \le t \le 1$

$$\boxed{f(x,y,z) = 5z^2e^{3x-2y} - 4x^3y - y^{2/3} + 3z, \quad \int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = -99 - 3^{2/3} - 80e^{-2}}$$